Generalized Analytical Predictor

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A generalized analytical predictor (GAP) approach (Wong and Seborg, 1984, 1986) has been proposed to enable the use of any feedback controller with the standard analytical predictor. In this Wong-Seborg GAP (WS-GAP), a disturbance predictor is included to predict future values of the load disturbance effect using the difference between the outputs of the actual process and the process model. The derivation of the disturbance predictor is based on the assumption that the disturbance transfer function is the same as the process transfer function and the process model is known exactly.

Wellons and Edgar (1985) have modified the WS-GAP to include a generalized disturbance predictor. The structure of the Wellons-Edgar GAP (WE-GAP) is shown to contain the WS-GAP. In the development of the WE-GAP, a general second-order disturbance transfer function is used. Assuming a first-order disturbance transfer function to be the process model and using a deadbeat filter, the WE-GAP reduces to the WS-GAP.

In this note, an effort is made to unify the two different derivations of the GAP from the viewpoint of parameter estimation. No assumptions on the disturbance transfer function made in the generalized analytical predictors are used in the present derivation. The assumption made here is believed to be more practical, and has led to the use of autoregressive model for predicting the future effects of load disturbances.

Parameter Estimation and Bias Term

In parameter adaptive control, the controlled process is usually approximated as a linear discrete equation with its parameters estimated recursively. The discrete equation for a first-order process is

$$y(k) = a_1 y(k-1) + b_1 u(k-1) \quad (1)$$

The u and y are deviation variables, that is,

$$y(k) = Y(k) - Y_{ss} \quad (2)$$

$$u(k) = U(k) - U_{ss} \quad (3)$$

where Y_{ss} and U_{ss} are steady state values. Since $u(k)$ and $y(k)$ are used in parameter estimation, the steady state values Y_{ss} and U_{ss} are required. However, these steady state values are not always available, especially for a nonlinear process experiencing frequent changes in operating conditions. This difficulty can be overcome by including a constant term in the discrete equation. Substituting Eqs. 2 and 3 into Eq. 1 gives

$$Y(k) = a_1 Y(k-1) + b_1 U(k-1) + d(k) \quad (4)$$

with

$$d(k) = (1 - a_1) Y_{ss} - b_1 U_{ss} \quad (5)$$

Thus, with the addition of the bias term $d(k)$, the actual measurements of process input and output can be used directly without knowing the steady state values.

Furthermore, this bias term has another important function. In the process industries, unmeasured load disturbances are quite common occurrences. These unmeasured disturbances are actually a part of process inputs that is not accounted for in the discrete process model. When a single-input/single-output (SISO) approach is adopted to obtain a "pure" relation between the desired input and output, a term for the unmeasured load disturbance must be included in the process model to prevent the model parameters from being estimated erroneously due to the additional contribution of the unmeasured disturbance. This can be achieved by assuming that the overall effect of all unmeasured disturbances on the process output can be represented by an equivalent term that experiences a sequence of step changes. Let this term be $L(k)$; we have

$$Y(k) = Y_d(k) - L(k) \quad (6)$$

where $Y(k)$ is the corrupted process output. Substituting Eq. 6 into Eq. 4 gives

$$Y_d(k) = a_1 Y_d(k-1) + b_1 U(k-1) + d(k) \quad (7)$$

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with

$$d(k) = (1 - a_1)Y_{ss} - b_1U_{ss} + L(k) - a_1L(k-1)$$
$$= c + L(k) - a_1L(k-1) \quad (8)$$

where

$$c = (1 - a_1)Y_{ss} - b_1U_{ss} \quad (9)$$

Disturbance Prediction

Suppose Y_{ss} and U_{ss} are known or can be measured; then the unmeasured disturbance $L(k)$ can actually be determined from the bias term as follows (later we will see that this assumption can be alleviated for the purpose of prediction):

$$L(k) = d(k) + a_1L(k-1) - c \quad (10)$$

This equation can then be used to recursively predict future values of the load disturbance effect:

$$L(k+1) = d(k+1) + a_1L(k) - c \quad (11)$$

$$\begin{aligned} L(k+2) &= d(k+2) + a_1L(k+1) - c \\ &= d(k+2) + a_1[d(k+1) \\ &\quad + a_1L(k) - c] - c \\ &= d(k+2) + a_1d(k+1) \\ &\quad + a_1^2L(k) - (1 + a_1)c \end{aligned} \quad (12)$$

$$\begin{aligned} L(k+3) &= d(k+3) + a_1L(k+2) - c \\ &= \cdots = d(k+3) + a_1d(k+2) \\ &\quad + a_1^2d(k+1) + a_1^3L(k) \\ &\quad - (1 + a_1 + a_1^2)c \end{aligned} \quad (13)$$

$$\vdots$$

$$\begin{aligned} L(k+n) &= [d(k+n) + a_1d(k+n-1) \\ &\quad + \cdots + a_1^{n-1}d(k+1)] \\ &\quad + a^nL(k - (1 + a_1 + \cdots + a^{n-1})c \end{aligned} \quad (14)$$

Note that future values of the bias term are needed in predicting the future load disturbance. They are unknown at present, however. Assuming that

$$d(k+n) = d(k+n-1) = \cdots = d(k) \quad (15)$$

Eq. 14 reduces to

$$\begin{aligned} &L(k+n) \\ &= (1 + a_1 + \cdots + a_1^{n-1})[d(k) - c] + a_1^nL(k) \end{aligned} \quad (16)$$

Since from Eq. 8

$$d(k) - c = L(k) - a_1L(k-1) \quad (17)$$

Eq. 16 becomes

$$\begin{aligned} L(k+n) &= (1 + a_1 + \cdots + a_1^{n-1}) \\ &\quad \cdot [L(k) - a_1L(k-1)] + a_1^nL(k) \\ &= (1 + a_1 + \cdots + a_1^{n-1})(1 - a_1z^{-1})L(k) + a_1^nL(k) \end{aligned} \quad (18)$$

Therefore, the prediction transfer function is

$$\begin{aligned} \frac{L(k+n)}{L(k)} &= (1 + a_1 + \cdots + a_1^{n-1})(1 - a_1z^{-1}) + a_1^n \\ &= \frac{1 - a_1^n}{1 - a_1}(1 - a_1z^{-1}) + a_1^n \end{aligned} \quad (19)$$

This is exactly the same as the disturbance predictor used in the WS-GAP. It should be noted that no assumption about the load disturbance transfer function used in the GAP is used in the derivation. Instead, the assumption made in Eq. 15 implies a first-order autoregressive model for predicting the future effects of the unmeasurable load disturbances on the process output. This implied assumption is more practical since no information about the actual load disturbance process is required. Moreover, an estimate of the current disturbance $L(k)$ can be obtained by subtracting the process model output from the actual process output, as done in the internal model control. As a result, the bias term is not needed in the prediction.

If, instead of a first-order transfer function model we have a second-order model for the process:

$$\begin{aligned} Y_d(k) &= a_1Y_d(k-1) + a_2Y_d(k-2) \\ &\quad + b_1U(k-1) + b_2U(k-2) + d(k) \end{aligned} \quad (20)$$

with

$$\begin{aligned} d(k) &= (1 - a_1 - a_2)Y_{ss} - (b_1 + b_2)U_{ss} \\ &\quad + [L(k) - a_1L(k-1) - a_2L(k-2)] \\ &= c + [L(k) - a_1L(k-1) - a_2L(k-2)] \end{aligned} \quad (21)$$

then the prediction can be achieved as follows:

$$\begin{aligned} L(k+n) &= d(k+n) + a_1L(k+n-1) \\ &\quad + a_2L(k+n-2) - c \\ &= d(k+n) + a_1[d(k+n-1) \\ &\quad + a_1L(k+n-2) \\ &\quad + a_2L(k+n-3) - c] \\ &\quad + a_2L(k+n-2) - c \\ &= d(k+n) + a_1d(k+n-1) \\ &\quad + (a_1^2 + a_2)L(k+n-2) \\ &\quad + a_1a_2L(k+n-3) - (1 + a_1)c \end{aligned} \quad (22)$$

$$\begin{aligned} &= \cdots = d(k+n) + a_1d(k+n-1) \\ &\quad + (a_1^2 + a_2)d(k+n-2) \\ &\quad + [a_1(a_1^2 + a_2) + a_1a_2]L(k+n-3) \\ &\quad + (a_1^2 + a_2)a_2L(k+n-4) \\ &\quad - [1 + a_1 + (a_1^2 + a_2)]c \end{aligned} \quad (23)$$

$$\vdots$$

$$= d(k+n) + A_{11}d(k+n-1) + \cdots + A_{n-1}d(k+1) + A_{n1}L(k) + A_{n2}L(k-1) - \left(1 + \sum_{i=1}^{n-1} A_{i1}\right)c \quad (24)$$

where A_{ij} are defined as

$$A_{11} = a_1, \quad A_{12} = a_2 \quad (25)$$

and

$$A_{i1} = a_1 A_{(i-1)1} + A_{(i-1)2} \quad (26)$$

$$A_{i2} = a_2 A_{(i-1)1}$$

If we assume

$$d(k+n) = d(k+n-1) = \cdots = d(k+1) = d(k) \quad (27)$$

Eq. 24 becomes

$$L(k+n) = \left(1 + \sum_{i=1}^{n-1} A_{i1}\right)[d(k) - c] + A_{n1}L(k) + A_{n2}L(k-1) \quad (28)$$

Because from Eq. 21

$$d(k) - c = L(k) - a_1 L(k-1) - a_2 L(k-2) = (1 - a_1 z^{-1} - a_2 z^{-2})L(k) \quad (29)$$

Eq. 28 becomes

$$L(k+n) = \left(1 + \sum_{i=1}^{n-1} A_{i1}\right)(1 - a_1 z^{-1} - a_2 z^{-2})L(k) + (A_{n1} + A_{n2}z^{-1})L(k) \quad (30)$$

The prediction transfer function is, therefore,

$$\frac{L(k+n)}{L(k)} = \left(1 + \sum_{i=1}^{n-1} A_{i1}\right)(1 - a_1 z^{-1} - a_2 z^{-2}) + (A_{n1} + A_{n2}z^{-1}) \quad (31)$$

This is structurally the same as the generalized disturbance predictor of the WE-GAP with a deadbeat filter. Again, no assumption made in the WE-GAP is made about the load disturbance transfer function. Similarly, the assumption of Eq. 27 results in a second-order autoregressive model for the future effects of the unmeasured disturbances. This analysis has led to the use of an autoregressive model-based predictor for future disturbance effects to improve the regulatory performance of an adaptive inferential control (Shen and Lee, 1987).

Literature Cited

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